Professur
Allgemeine Nachrichtentechnik

Prof. Dr.-Ing. Udo Zölzer

HELMUT SCHMIDT UNIVERSITÄT

Nonlinear-Phase Basis Functions in Quasi-Bandlimited Oscillator Algorithms

Jussi Pekonen[1] and Martin Holters[2]

[1]Independent, http://www.pekonen.cc/
[2]Department of Signal Processing and Communications, Helmut Schmidt University

September 19, 2012

Classical Waveforms

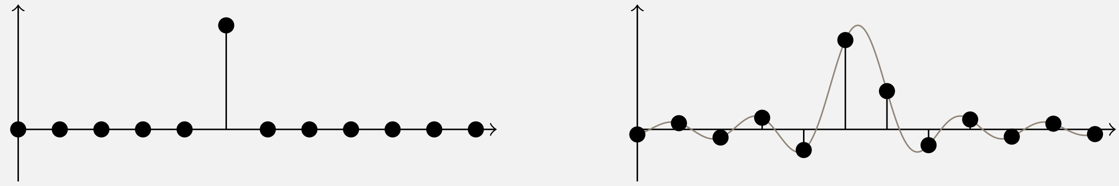
HELMUT SCHMIDT UNIVERSITÄT

- Subtractive synthesis is a rather common synthesis method
- A spectrally rich signal is filtered to obtain the desired harmonic content
- Typical oscillator waveforms: square, triangle, sawtooth
  - Relatively easy to build in analog hardware
  - Aliasing issue when generated digitally
  - Alias-reduced digital waveforms can be obtained from an impulse or unit step train

BLIT / BLEP

HELMUT SCHMIDT UNIVERSITÄT

- Digital generation of an impulse function is trivial at sampling instants
- Between sampling instants, sinc function gives correct solution



- In practice: Approximated either with a tabulated function (typically windowed sinc) or with a polynomial function $\Rightarrow$ BLIT
- Alternatively, same reasoning for unit steps gives BLEP method
- BLIT and BLEP generators are typically FIR filters
- Why not use IIR filters?

Modified Impulse-Invariant Transform

HELMUT SCHMIDT UNIVERSITÄT

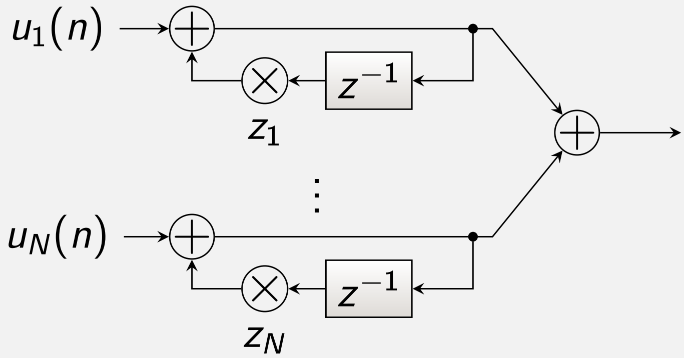
- Approach proposed here: Use recursive lowpass for band-limiting
- Start with suitable analog prototype lowpass $H(s)$
- Sample its impulse response at time instants $t = (n + d)T$
  - Sampling interval T
  - Offset d within the sampling interval, $0 \leq d < 1$
- Apply z-transform to get

$$H(z, d) = \sum_{k=1}^{N} \frac{r_k z_k^d}{1 - z_k z^{-1}}$$

  Filter order N and r_k and z_k depend on initial analog filter
- Note: Only the numerators depend on d

Resulting System

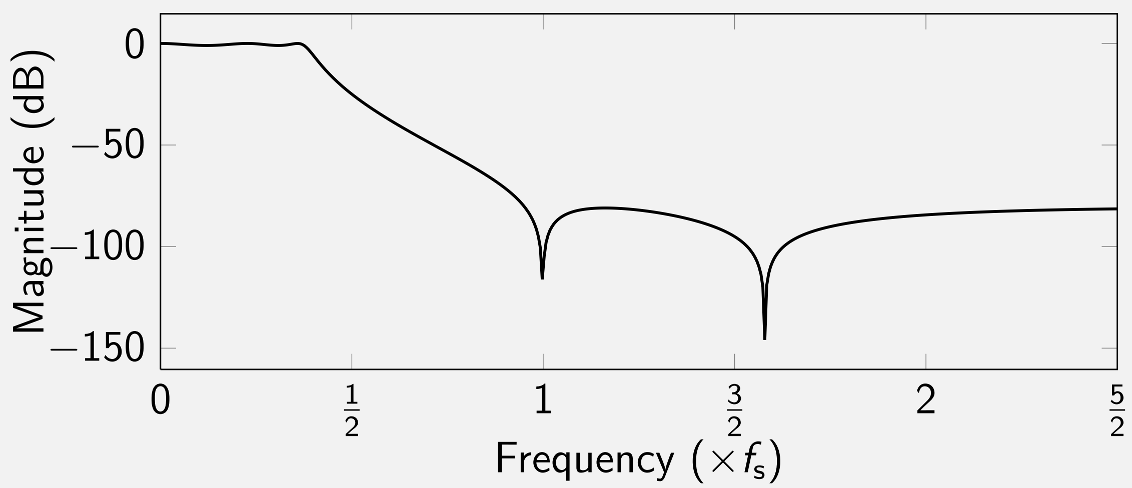
For a pulse start at $t_\mathrm{p} = (n_\mathrm{p} - d_\mathrm{p})T$ we excite the system



with $$u_k(n) = \begin{cases} r_k z_k^{d_\mathrm{p}} & \text{when } n = n_\mathrm{p} \\ 0 & \text{otherwise} \end{cases}$$

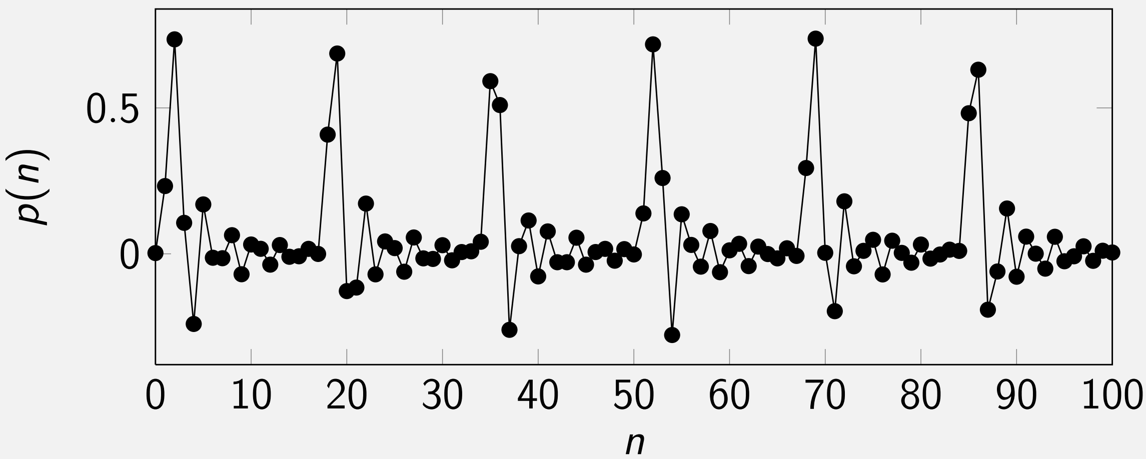
Example: Prototype Lowpass Filter

5th-order, cut-off frequency at $\frac{3}{8}f_\mathrm{s}$, passband ripple 1 dB, stop-band attenuation 81 dB



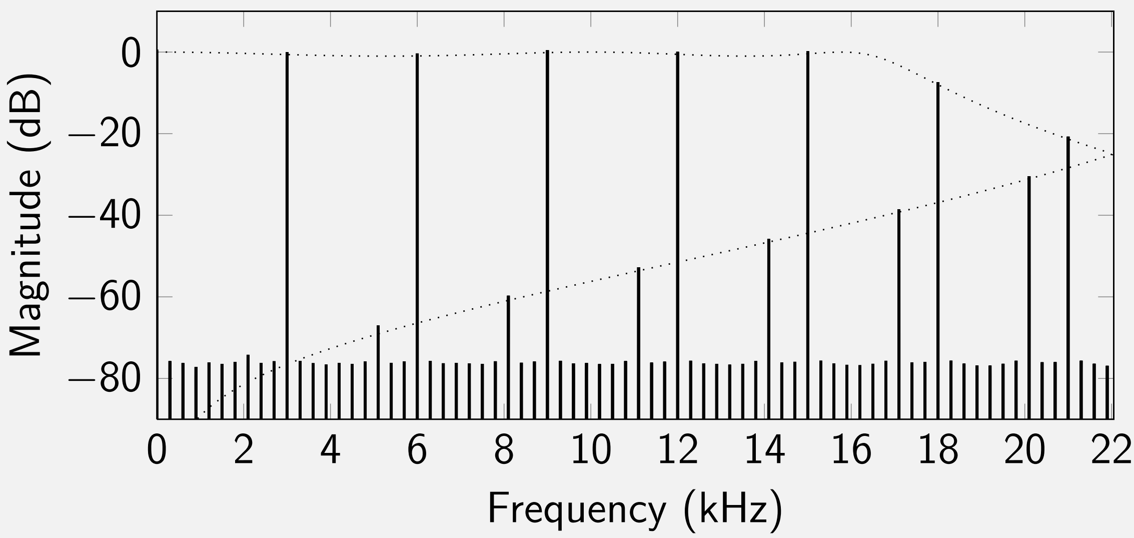
Example: Impulse Train

HELMUT SCHMIDT UNIVERSITÄT



Example: Resulting Spectrum

HELMUT SCHMIDT UNIVERSITÄT



Reducing Computational Complexity

- In general, the lowpass filter will have complex poles $\Rightarrow$ Complex-valued first-order subsystems
  - Tedious but straight-forward to combine conjugate complex first-order subsystems to real-valued second-order systems
  - Excitation for one pulse then spans two samples
- Excitation signals expensive to compute

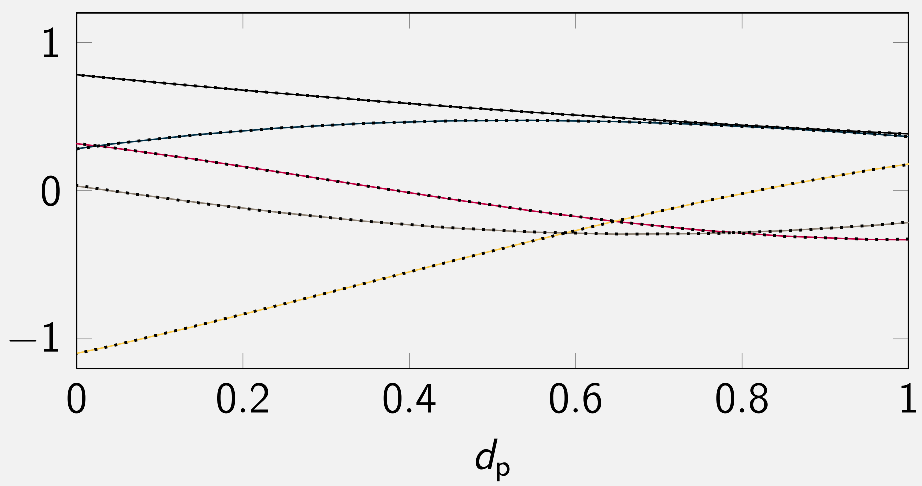
$$u_k(n) = \begin{cases} r_k z_k^{d_\mathrm{p}} & \text{when } n = n_\mathrm{p} \\ 0 & \text{otherwise} \end{cases}$$

(similar for real-valued system)

$\Rightarrow$ Polynomial approximation of the excitation signals

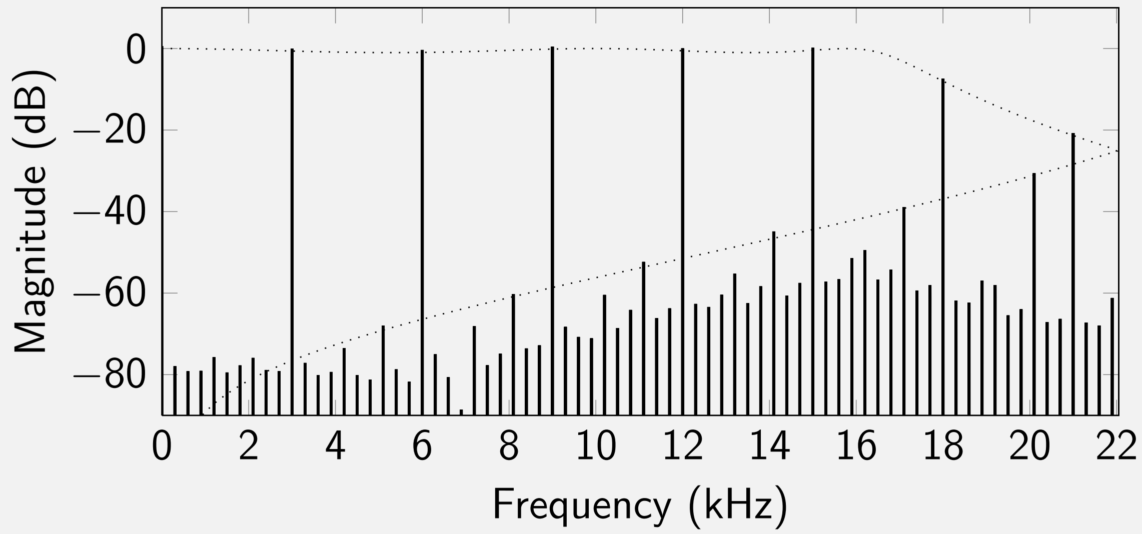
Example: Values of the Excitation

With 3rd-order polynomial approximation of the excitation signals



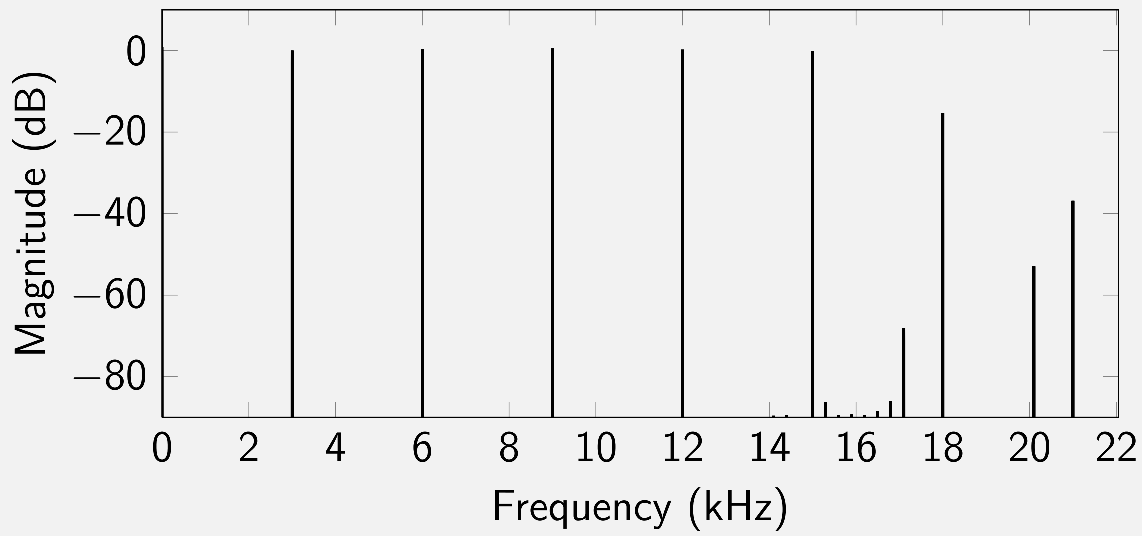
Example: Resulting Spectrum with Polynomial Approximation

5th-order lowpass filter, 3rd-order polynomial approximation



Example: Resulting Spectrum, Higher Orders

7th-order lowpass filter, 5th-order polynomial approximation



Listen to it!

HELMUT SCHMIDT UNIVERSITÄT

- Ideal rectangular pulse wave
- IIR-generated rectangular pulse wave, 3rd-order lowpass, 1st-order polynomial
- IIR-generated rectangular pulse wave, 5th-order lowpass, 3rd-order polynomial
- IIR-generated rectangular pulse wave, 7th-order lowpass, 5rd-order polynomial

Conclusions

HELMUT SCHMIDT UNIVERSITÄT

- Recursive filters leads to oscillators with nonlinear-phase basis functions
- Low memory requirements (no look-up tables)
- Trade-off between quality and computational complexity
- These slides, more sound examples, and implementations can be found at `http://pekonen.cc/p/dafx12-nlpo/`